Pool 3

THE ACTION

OF

JUPITER UPON COMET V, 1889,

BY

CHARLES LANE POOR, M. S.

A Thesis submitted to the Johns Hopkins University for the degree of Doctor of Philosophy.

APRIL, 1891.

BALTIMORE:
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INTRODUCTION.

On July 6th, 1889, at Geneva, New York, Brooks discovered a faint telescopic comet, since known as Comet d and V 1889. During the following summer and early fall few observations were made. Soon, however, the periodic character of this body was recognized, and when Mr. Searle pointed out that in 1886 it must have passed very close to Jupiter the interest of astronomers was at once aroused. Mr. Chandler of Boston verified this suggestion, and undertook the determination of the effect of this close approach. His results were published in the *Astronomical Journal*, No. 205, where he showed that the orbit had been radically changed, that before 1886 the comet was moving in an entirely different ellipse from that in which it is at present moving. Before any note on the subject had been published, Prof. Newcomb, being unaware that any one was then working on the problem, suggested it as an appropriate subject for a thesis. I undertook the work, made a series of observations, deduced new elements, and was busy computing the perturbations, when Mr. Chandler's first paper appeared. I at once laid the work aside, but after some months, with Mr. Chandler's assent, I again attacked the very fascinating problem.

At present the comet is moving in a small seven years' ellipse. Mr. Chandler found that before the encounter with Jupiter the comet had been moving in a large ellipse, with a twenty-seven years' period, whose perihelion was almost exactly the present aphelion distance. That is, the original orbit was not only much larger than the present one, but its position in space was different, the directions of the lines of apsides and nodes were nearly completely reversed, and the plane of the orbit was tilted a number of degrees. Again Mr. Chandler shows that, assuming the above changes to be substantially correct, there could have been no previous close approach to any planet, sufficient to appre-

ciably disturb the comet's orbit, back to the year 1779. But that in that year the comet again passed under the control of Jupiter and then experienced a radical change of orbit, and that at the same point of longitude when Lexell's comet underwent its notable disturbance in that year. Moreover the most probable elements of Lexell's comet subsequent to its disturbance and those of Comet V previous to 1886 show such a striking likeness, that one at once infers the identity of the two bodies.

Lexell's comet, it will be remembered, was discovered by Messier on the night of June 14–15, 1770. He at first took it to be a small nebula, and did not recognize its cometary character until two or three nights later. The comet was then rapidly approaching the earth; it became visible to the naked eye on June 21st, and on July 2nd, passed closer to the earth than any other known comet. It was then about as bright as the North Star, and seen through a telescope, its diameter was found to be about twice that of full moon, and it had a well marked, though small tail. On July 4th, it passed into the rays of the sun, and became invisible, only to reappear, however, on August 4th, and to remain visible to the naked eye until the 26th, and to the telescope until October 2nd.

Lexell was the first to point out that this comet was periodic, was then revolving about the sun in an ellipse of 5.58 years. To the objection that it had not been seen six years before, he proved that in 1767, it had been in conjunction with Jupiter, and that as their mutual distance was only $\frac{1}{580}$ that of the comet from the sun, the action of Jupiter had probably altered its orbit considerably. He also predicted a second close approach in 1779, and said that this might prevent its reappearance after that date. This prediction of Lexell's was fulfilled, for the comet was never again seen, unless, indeed, it prove that the comet discovered by Brooks on July 6, 1889, is this lost body.

About 1845, Le Verrier carefully and completely worked out the theory of this comet's movements. His conclusions were substantially the same as Lexell's. The main fact was perfectly clear; the comet had passed very close to Jupiter in 1779, perhaps had even passed in among the satellites of Jupiter. The resulting changes in the orbit were enor-

mous. But unfortunately it was found that the original observations were not exact enough to completely determine the orbit of the comet—various systems of elements would equally well represent the observations. The differences were small, but became magnified many times in the various orbits deduced for the comet after its appulse with Jupiter. Le Verrier expressed these different sets of elements in terms of an indeterminate quantity μ; the unit of which corresponds to an arbitrary change of 0.01 in the semi-major axis of the orbit that the comet had in 1770. He gives a table containing the resulting values of the elements for different assumed values of μ. On the hypothesis that $\mu=0$ the comet passed the centre of Jupiter at no greater distance than three and a half radii of that planet; the resulting orbit in this case would be an hyperbola and the comet would have vanished forever. Again, on the most probable hypothesis of $\mu=1$ according to Le Verrier, the comet passed Jupiter at a distance of one hundred of that planet's radii, and the resulting orbit was an ellipse of about nine years' period. Besides these extreme cases we have for different values of μ an immense variety of resulting orbits. Such were Le Verrier's conclusions:—the original data furnished no complete solution to the problem of what became of Lexell's comet after its encounter with Jupiter in 1779.

For the value of $\mu=0.35$ the resulting elements of Lexell's comet agree very well with those derived by Mr. Chandler for Comet V before its disturbance in 1886. Yet this agreement is by no means close enough to establish the identity of the two bodies. The presumption, however, in favor of such identity is very strong, as Mr. Chandler clearly points out. Again it must be remembered that his numerical results are but a first approximation. He takes account only of the principal perturbations, that is, the action of Jupiter is only considered during the few months of very close approach. With his assent I have carried his work a few steps farther, making a second approximation to the numerical solution of the problem.

While the numerical part of my work is to be regarded only as a second step, I have tried to make the methods pursued as complete as possible. Where new or rare formulæ are used the derivation of them

is shown and their use clearly explained. The numerical computations were made during the summer of 1890 and the results published, as a note, in *Astronomical Journal*, No. 228. Before the thesis was finished, however, some unexpected and very valuable observations of the comet were obtained at the Lick Observatory during the months of November and December, 1890, or nearly nine months after the regular series ended. These observations have been inserted into the original manuscript, and although the resulting changes are not very marked, yet they caused, practically, an entire re-computation of the numerical part of this work. The results thus obtained appear in the following pages and were published in *Astronomical Journal*, No. 244. They give, I think, a very good approximation to the definitive determination of the elements and of the character of the approach to Jupiter in 1886.

Note.—February, 1892.—I am now engaged in computing definitive elements for this interesting body. When that work is completed I shall carry the elements back to 1886 and determine as accurately as possible all the phenomena of the approach in that year.

The entire work may be conveniently divided into the following parts:

FIRST. The derivation of a set of elements that represent the motion of the comet about the sun at the moment when I chose to transpose Jupiter and the sun, as central force and disturbing force, respectively. This part includes the correction of a preliminary orbit by comparison with the observations, and a computation of the principal perturbations suffered by the comet between the time of quitting Jupiter in 1886, till the time of appearance in 1889. There is in this section nothing but ordinary routine astronomical work.

SECOND. The transformation of the centre of motion from the sun to Jupiter, and a computation of the solar perturbations during the time that Jupiter is regarded as the central force. This section contains the main part of the mathematical treatment of the problem, and the derivation and discussion of the various formulæ needed.

THIRD. The transformation from Jupiter to the sun as the central force, and the computation of the perturbations by Jupiter for a few months before the appulse. This section is similar in character to the second, but the mathematical treatment is less extensive.

FOURTH. Discussion of the results and a comparison of the final orbit obtained with Le Verrier's determination of the orbit of Lexell's comet.

SECTION FIRST.

CORRECTION OF THE PRELIMINARY ORBIT AND COMPUTATION OF THE PERTURBATIONS.

I. Correction of the approximate elements of the comet's orbit.

The elements given by Mr. Chandler in the *Astronomical Journal*, No. 205, were used as the approximate elements to be corrected by the method of the "Variation of two Geocentric Distances." These elements are:

$$T = 1889,\ \text{Sept. } 30.0119 \text{ Gr. M. T.}$$

$$\left.\begin{array}{l} \pi = 1^\circ\ 26'\ 17.3'' \\ \Omega = 17\ \ 58\ \ 45.3 \\ i = 6\ \ \ 4\ \ 10.5 \end{array}\right\} 1890.0$$

$$e = 0.470704$$

$$a = 3.684682$$

$$q = 1.950229$$

Period 7.0730 years.

With the above elements, a partial ephemeris was computed and ten normal places formed. Each place was formed from the observations made on not more than three days and observations from the large observatories only were used. The places were corrected for aberration and parallax with the following results:

1889.	RIGHT ASCENSION. h. m. s.	DECLINATION. ° ′ ″	WEIGHT.	OBSERVATORY.
July 9.5	23 46 59.47	− 8 52 34.0	3	Mt. H.
Aug. 1.5	0 4 26.61	− 7 0 27.1	3	Munich & Hamburg.
Sept. 27.5	23 49 42.40	− 5 12 13.0	2	Washington.
Oct. 18.5	23 40 44.45	− 3 54 34.5	3	Wash. & Balto.
Nov. 15.5	23 47 49.95	− 0 40 11.7	3	Princeton.
Dec. 22.5	0 27 4.95	+ 5 29 12.9	3	Washington.

1890.	RIGHT ASCENSION. h. m. s.	DECLINATION. ° ′ ″	WEIGHT.	OBSERVATORY.
Jan. 13.5	1 0 11.94	+ 9 35 46.5	3	Princeton.
Feb. 14.5	1 55 20.32	+15 27 43.9	3	Princeton.
Nov. 22.0	9 0 34.28	+24 13 13.1	1	Mt. H.
Dec. 21.0	8 54 10.20	+25 25 1.70	1	Mt. H.

The above right ascensions and declinations, which refer to the apparent equinox and equator, were then reduced to the mean equinox and equator of the beginning of the years 1889 and 1890 respectively.

Assuming as correct, $\log \rho = 0.1560744$ and $\log \rho' = 0.3922000$ which are the values of the geocentric distances for July 9.5 and Feb. 14.5 respectively, I have, for the complete geocentric positions of the comet on these dates:

July 9.5

$$\left.\begin{aligned} \alpha &= -3^\circ\ 15'\ 16.8'' \\ \delta &= -8\ \ 52\ \ 38.1 \end{aligned}\right\} 1889.0$$

$$\log \rho = 0.1560744$$

Feb. 14.5

$$\left.\begin{aligned} \alpha &= +28^\circ\ 50'\ 14.4'' \\ \delta &= +15\ \ 27\ \ 47.2 \end{aligned}\right\} 1890.0$$

$$\log \rho' = 0.3922000$$

Transforming these geocentric positions into the corresponding heliocentric positions and then referring the second place to the mean equinox and equator of 1889.0, I have:

July 9.5

$$\lambda = -33^\circ\ 12'\ 29.5''$$
$$\beta = -\ 4\ \ 44\ \ 4.0$$
$$\log r = 0.3157274$$

Feb. 14.5

$$\lambda = +55^\circ\ 52'\ 57.7''$$
$$\beta = +\ 3\ \ 44\ \ 21.2$$
$$\log r' = 0.3528720$$

From these a set of elements was computed which exactly represents the two observed places. These elements are elsewhere designated as belonging to hypothesis (°). An increment $\Delta\rho = -0.001$, a change of unity in the third place of the logarithm, was then assigned to ρ, and with the geocentric distances $\rho + \Delta\rho$ and ρ', was computed a second set of elements designated as ($'$). Next, a similar increment $\Delta\rho' = -0.001$, was assigned to ρ', and from ρ and $\rho' + \Delta\rho'$ a third set of elements, designated as ($''$), computed. These sets of elements, each of which exactly represents the two normal places of July 9.5 and Feb. 14.5, are:

	(°)	(′)	(″)
π	1° 28′ 7″.24	1° 16′ 7″.62	1° 43′ 27″.88
Ω	17 57 17.46	17 54 53.39	17 59 24.92
i	6 4 9.00	6 3 49.60	6 3 58.23
log e	9.6735694	9.6724582	9.6708118
" a	0.5670766	0.5657482	0.5643681
" μ	2.6993917	2.7013843	2.7034544
T 1889	Sept. 30.08039	Sept. 29.603867	Sept. 30.73025

The right ascension and declination for the date of each intermediate normal were then computed from each set of elements. The results, as well as the observed right ascension and declination, are exhibited in the following table; where they are reduced to the mean equator and equinox of 1889 and 1890 respectively:

		OBSERVED.	HYP. °.	HYP. ′.	HYP. ″.
1889.		h. m. s.	h. m. s.	h. m. s.	h. m. s.
Aug. 1.5	α	0 4 23.79	0 4 25.05	0 4 27.11	0 4 22.09
	δ	− 7° 0′ 32″.5	− 7° 0′ 16″.4	− 6° 59′ 57″.2	− 7° 0′ 51″.4
Sept. 27.5		23 49 41.25	23 49 44.87	23 49 45.84	23 49 31.78
		− 5 12 24.5	− 5 11 23.0	− 5 10 37.4	− 5 13 40.40
Oct. 18.5		23 40 43.10	23 40 48.73	23 40 49.31	23 40 33.23
		− 3 54 43.3	− 3 53 54.58	− 3 53 3.2	− 3 56 19.7
Nov. 15.5		23 47 48.45	23 47 53.24	23 47 54.34	23 47 37.93
		− 0 40 22.0	− 0 39 28.2	− 0 38 46.0	− 0 41 45.1
Dec. 22.5		0 27 2.98	0 27 6.56	0 27 7.94	0 26 55.69
		+ 5 29 0.3	+ 5 29 37.6	+ 5 30 4.9	+ 5 28 10.2
1890.					
Jan. 13.5		1 0 12.94	1 0 14.68	1 0 15.71	1 0 7.73
		+ 9 35 52.2	+ 9 36 9.1	+ 9 36 24.80	+ 9 35 17.6
Nov. 22.0		9 0 32.35	8 59 58.43	8 59 52.85	9 1 43.08
		+ 24 13 19.2	+ 24 15 44.70	+ 24 15 50.50	+ 24 7 29.80
Dec. 21.0		8 54 7.78	8 53 30.07	8 53 23.54	8 55 32.16
		+ 25 25 9.7	+ 25 28 10.0	+ 25 28 16.6	+ 25 19 17.3

Then for each intermediate normal we have:

$$\cos\delta\Delta\alpha = \cos\delta\frac{da}{d\rho}\Delta\rho + \cos\delta\frac{da}{d\rho'}\Delta\rho'$$

$$\Delta\delta = \frac{d\delta}{d\rho}\Delta\rho + \frac{d\delta}{d\rho'}\Delta\rho'$$

Where,

$$\Delta\alpha \equiv \alpha - \alpha^\circ,\quad \frac{da}{d\rho} \equiv \alpha' - \alpha^\circ,\quad \frac{da}{d\rho'} \equiv \alpha'' - \alpha^\circ$$

$$\Delta\delta \equiv \delta - \delta^\circ,\quad \frac{d\delta}{d\rho} \equiv \delta' - \delta^\circ,\quad \frac{d\delta}{d\rho'} \equiv \delta'' - \delta^\circ$$

Whence we have the equations of condition to determine $\Delta\rho$ and $\Delta\rho'$;

FOR RIGHT ASCENSION.

				RESIDUALS.	WEIGHT.
Aug.	1.5	$-18''.90 = +30''.90\Delta\rho$	$-\ 44''.40\ \Delta\rho'$	$-\ 0''.5$	3
Sept.	27.5	$-\ 54.30 = +14.55$	$-\ 196.35$	$+\ 8.6$	2
Oct.	18.5	$-\ 84.45 = +\ 8.70$	$-\ 232.50$	$-\ 1.1$	3
Nov.	15.5	$-\ 71.85 = +16.50$	$-\ 229.65$	$+\ 2.8$	3
Dec.	22.5	$-\ 53.70 = +20.70$	$-\ 163.05$	$-\ 0.1$	3
Jan.	13.5	$-\ 27.60 = +15.45$	$-\ 104.25$	$+\ 7.0$	3
Nov.	22.0	$+508.80 = -86.20$	$+1569.50$	$+\ 9.8$	1
Dec.	21.0	$+565.65 = -98.07$	$+1831.25$	$-\ 6.3$	1

FOR DECLINATION.

				RESIDUALS.	WEIGHT.
Aug.	1.5	$-16''.10 = +19''.20\Delta\rho$	$-\ 35''.00\ \Delta\rho'$	$-\ 2''.4$	3
Sept.	27.5	$-\ 61.50 = +45.60$	$-\ 137.40$	-11.0	2
Oct.	18.5	$-\ 48.70 = +51.40$	$-\ 145.10$	$+\ 4.1$	3
Nov.	15.5	$-\ 53.80 = +42.20$	$-\ 136.90$	$-\ 5.0$	3
Dec.	22.5	$-\ 37.30 = +27.60$	$-\ 87.40$	$-\ 6.0$	3
Jan.	13.5	$-\ 16.90 = +15.70$	$-\ 51.56$	$+\ 1.3$	3
Nov.	22.0	$-145.46 = +\ 5.80$	$-\ 494.90$	$+\ 9.2$	1
Dec.	21.0	$-180.30 = +\ 6.60$	$-\ 532.70$	-14.3	1

Solving these equations by the method of least squares:

$$\Delta\rho = -0.150831$$
$$\Delta\rho' = +0.309693$$

in negative units of the third decimal of the logarithm. Hence,

$$\log \rho = 0.1562252$$
$$\log \rho' = 0.3918903$$

and the complete geocentric positions become,

$$\begin{array}{lll} \text{July } 9.5 & & \text{Feb. } 14.5 \\ \left.\begin{array}{l} \alpha = -3^\circ\ 15'\ 16''.8 \\ \delta = -8\ \ 52\ \ 31.1 \end{array}\right\} 1889.0 & & \left.\begin{array}{l} +28^\circ\ 50'\ 14''.4 \\ +15\ \ 27\ \ 47.2 \end{array}\right\} 1890.0 \\ \log \rho = 0.1562252 & & \log \rho' = 0.3918903 \end{array}$$

From these corrected geocentric positions the final elements were computed, and the positions for the intermediate dates, as derived from them, compared with the original normal places. The resulting residuals (observed—computed), while not as small as could be wished for, still show that the elements are exact enough for my purpose.

These elements are:

$$\left.\begin{array}{l} \pi = \ \ 1^\circ\ 35'\ 31''.53 \\ \Omega = 17\ \ 59\ \ 32.97 \\ i = \ \ 6\ \ \ 4\ \ 13.18 \end{array}\right\} 1890.0$$

$$\begin{array}{l} \log a = 0.5664512 \\ \text{“} \quad e = 9.6729017 \\ \text{“} \quad \mu = 2.7003308 \\ T = 1889 \text{ Sept. } 30.355026 \text{ Gr. M. T.} \end{array}$$

II. Perturbations by Jupiter from October 1886 to July 1889.

Considering the above elements as osculatory for July 2nd, 1889, I computed the perturbations by Jupiter from that date to the time of appulse in October 1886. The ordinary method of the "Variation of Constants" as given by Oppolzer was followed. Until March 15th, 1887, an interval of forty days was used. At this date the perturbations were integrated and applied to the elements. With the osculating elements thus derived for March 15th, 1887, the perturbations were computed until October 26th, 1886, using an interval of ten days, when they were again integrated and applied to the elements. In order to take account

of quantities of the second order in computing the differential coefficients, the variable elements were used in the computation for the various dates between March 1887 and October 1886.

The values of the differential coefficients and their summation are tabulated for the various elements. These tables are inserted at the end of the thesis. From these the integrated perturbations were obtained by the use of the formulæ for mechanical integration as follows:

$$'f = \frac{1}{12} f_0' - \frac{11}{720} f_0'''$$

$$''f = -\frac{1}{12} f_0 + \frac{1}{240} f_0''$$

$$u_0 = 'f - \frac{1}{12} f_0' + \frac{11}{720} f_0'''$$

$$u_0^2 = ''f + \frac{1}{12} f_0 - \frac{1}{240} f_0''$$

The elements thus derived for October 26.0, the date chosen to transfer the centre of motion to Jupiter, are as follows:

$$\left.\begin{array}{rl} L = & 215^\circ\ 47'\ 0''.6 \\ \pi = & 2\ \ 37\ \ 7.1 \\ \Omega = & 19\ \ \ 6\ \ 36.3 \\ i = & 7\ \ 23\ \ 37.2 \end{array}\right\} 1890.0$$

$$\begin{array}{rl} \log\ a = & 0.5550265 \\ \text{"}\ \ e = & 9.7209785 \\ \text{"}\ \ \mu = & 2.7174669 \end{array}$$

PART SECOND.

TRANSFORMATION OF THE CENTRE OF MOTION FROM THE SUN TO JUPITER.

La Place, in the fourth volume of the *Mécanique Céleste*, developes a method for determining the perturbations of a comet, when approaching very near a planet. This method, first proposed by D'Alembert, consists in supposing the planet to have a sphere of activity, within which the relative motion of the comet is affected only by the planet's attraction and beyond which the absolute motion of the comet about the sun is performed as if the sun alone acted upon it. The radius of this sphere depends upon the mass of the planet and its distance from the sun. This method is very simple and beautiful, but it neglects entirely the effect of the sun as a disturbing body whilst the comet is traversing its relative orbit about the planet. It will become much more effective, if we merely use the idea of the sphere of activity as defining approximately the point at which we may conveniently transpose the sun and the planet, as disturbing force and central force respectively; and after the transformation has been made, we may treat the sun and the comet as bodies revolving around the planet as a central body; the sun acting as a disturbing body upon the comet. The perturbations of the comet by the sun may be computed in a manner entirely similar to the usual methods. The exact point in the comet's orbit at which the transformation is made is of no great importance, provided that the perturbations be carefully computed both before and after. The most convenient point will be that given by La Place's idea of the sphere of activity.

To obtain the radius of this sphere La Place proceeds as follows: Let r and r' be the radii vectores of comet and the planet respectively, then the sun's action on the comet will be proportional to $\frac{1}{r^2}$, and that of the

planet upon the comet proportional to $\frac{m'}{(r'-r)^2}$. When the comet is without the sphere of activity of the planet, the quantity $\frac{1}{r^2}$ must greatly exceed $\frac{m'}{(r'-r)^2}$. Within this sphere the disturbing action of the sun on the comet, which is proportional to $\frac{1}{r^2} - \frac{1}{r'^2}$ or very nearly to $\frac{2(r'-r)}{r^3}$ must be very small in comparison to $\frac{m'}{(r'-r)^2}$. These two conditions are satisfied on the supposition that $\frac{m'}{(r'-r)^2}$ is a mean proportional between $\frac{1}{r^2}$ and $\frac{2(r'-r)}{r^3}$ and this gives for the radius $r'-r=\rho$ of the sphere of activity

$$\rho = r^5 \sqrt{\frac{1}{2} m'^2}$$

For the planet Jupiter I found,

$$\log \frac{\rho}{r} = 8.73166$$

And that day on which the values of ρ and r most nearly satisfy this equation will be the most convenient time for the transformation. This day I found to be October 26.0, 1886, at which time we have,

$$\begin{aligned} \log\ \rho &= 9.45802 \\ \text{"}\quad r &= 0.72762 \\ \text{"}\quad \frac{\rho}{r} &= 8.73040 \end{aligned}$$

Having thus determined upon the time of change, and having derived the elements about the sun for that instant, the process is to find from these elements the values of x, y, z, D_tx, D_ty, and D_tz, for the comet referred to the sun. Then from the Ephemeris, or tables, we find the values of the corresponding quantities x', y', z', D_tx', D_ty', D_tz', for Jupiter referred to the sun. Whence for the relative coördinates of the comet referred to Jupiter as a centre we have,

$$\begin{aligned} x_1 &= x - x' & D_tx_1 &= D_tx - D_tx' \\ y_1 &= y - y' & D_ty_1 &= D_ty - D_ty' \\ z_1 &= z - z' & D_tz_1 &= D_tz - D_tz' \end{aligned}$$

Then from these quantities we can determine the elements of the relative orbit about Jupiter.

FIRST. To find the values of the relative coördinates and their differential coefficients.

The values of x, y, and z may be obtained directly from the elements by the usual formulae, and the values of the differential coefficients, $D_t x$, $D_t y$, and $D_t z$ by the corresponding differential formulae. Assuming the ecliptic as the fundamental plane, with the positive direction of the axis of x directed towards the vernal equinox, we have, following the notation used by Watson in his Theoretical Astronomy,

$$x = r \sin a \sin (A + u)$$
$$y = r \sin b \sin (B + u)$$
$$z = r \sin i \sin u$$

Where the auxiliary quantities, a, b, A and B are the constants for the ecliptic, and are functions of Ω and i alone. They are determined by the formulae:

$$\cot a = -\tan \Omega \cos i; \ \cot B = \cos \Omega \cos i$$
$$\sin a = \frac{\cos \Omega}{\sin A} \qquad ; \ \sin b = \frac{\sin \Omega}{\sin B}$$

Differentiating the above expressions for x, y and z, with regard to the time we have,

$$D_t x = \frac{x}{r} D_t r + r \sin a \cos (A + u) D_t u$$
$$D_t y = \frac{y}{r} D_t r + r \sin b \cos (B + u) D_t u$$
$$D_t z = \frac{z}{r} D_t r + r \sin i \cos u \ D_t u$$

The values of $D_t r$ and $D_t u$ may be found from the equations:

$$r^2 D_t v = a^2 \cos \phi \ D_t M$$
$$D_t r = a \tan \phi \sin v \ D_t M$$

where

$$D_t M = n = \mu$$

and

$$D_t u = D_t v$$

Thus from the elements of the comet for October 26.0 as before given, I find for that date,

$$v = 192^\circ\ 19'\ 5''.43$$
$$u = 175\quad 49\ 36.23$$
$$\log\ r = 0.7276229$$
$$\text{"}\ D_t u = 6.9875183$$
$$\text{"}\ D_t r = 7.0784501\ n$$

And from these with the values of x, y and z deduced directly from the elements, I find,

$$\log\ x = 0.7126056\ n$$
$$\text{"}\ y = 0.1397898\ n$$
$$\text{"}\ z = 8.6991237$$
$$\text{"}\ D_t x = 7.3945983$$
$$\text{"}\ D_t y = 7.6687772\ n$$
$$\text{"}\ D_t z = 6.8307732\ n$$

From the *British Nautical Almanac* for 1886, after reducing the latitude and longitude to the mean equinox and equator of 1890.0, I find for the position of Jupiter on October 26.0, 1886,

$$\lambda' = 197^\circ\ 39'\ 36''.2$$
$$\beta' = \quad 1\ 17\ 45.6$$
$$\log r' = 0.7368541$$

To find the differential coefficients of these quantities I took from the *Almanac* seven complete positions of the planet separated by intervals of twenty days and reduced each to the mean fixed equinox of October 26.0 by applying the corrections for precession and nutation. I then tabulated the longitudes and found the differences to the third and fourth orders, and from these by the formulae of interpolation I found the first difference, or the $D_t\lambda$, for October 26.0. Similar processes gave me the $D_t\beta$ and $D_t r$,

$$\Delta\lambda' = +0^\circ\ 4'\ 31''.81$$
$$\Delta\beta' = -0\ \ 0\ \ 0.95$$
$$\Delta \log r' = -0.000002545$$

Reducing the $D_t\lambda$ and $D_t\beta$ to radians, I have for the differential coefficients with respect to the respect to the time,

$$\begin{aligned} \log D_t\lambda' &= 7.1198403 \\ \text{“}\quad D_t\beta' &= 4.6632985\, n \\ \text{“}\quad D_tr' &= 5.5047041\, n \end{aligned}$$

To find x', y', z' and their differential coefficients, we have the ordinary formulae,

$$\begin{aligned} x' &= r' \cos \beta' \cos \lambda' \\ y' &= r' \cos \beta' \sin \lambda' \\ z' &= r' \sin \beta' \end{aligned}$$

and the corresponding differential formulae,

$$\begin{aligned} D_tx' &= \frac{x'}{r'} D_tr' - y' D_t\lambda' - z' \cos \lambda' D_t\beta' \\ D_ty' &= \frac{y'}{r'} D_tr' + x' D_t\lambda' - z' \sin \lambda' D_t\beta' \\ D_tz' &= \sin \beta' D_tr' + r' \cos \beta' D_t\beta' \end{aligned}$$

Substituting the values already given I find for the rectangular coördinates of Jupiter referred to the sun, and their differential coefficients,

$$\begin{aligned} \log\quad x' &= 0.7157782\, n \\ \text{“}\quad y' &= 0.2187138\, n \\ \text{“}\quad z' &= 9.0912994 \\ \text{“}\quad D_tx' &= 7.3444712 \\ \text{“}\quad D_ty' &= 7.8350142\, n \\ \text{“}\quad D_tz' &= 5.4123645\, n \end{aligned}$$

From these with the values already given for the rectangular coördinates of the comet referred to the sun, I find for the relative coördinates of the comet, and their differential coefficients, referred to Jupiter as a centre, the following,

$$\begin{aligned} \log\quad x_1 &= 8.5778238 \\ \text{“}\quad y_1 &= 9.4392746 \\ \text{“}\quad z_1 &= 8.8655648 \\ \text{“}\quad D_tx_1 &= 6.4320641 \\ \text{“}\quad D_ty_1 &= 7.3374871 \\ \text{“}\quad D_tz_1 &= 6.8138767\, n \end{aligned}$$

SECOND. From the above relative coördinates and their differential coefficients to find the relative orbit of the comet about Jupiter.

The following integrals result from the equations of motion of one body around another (*Méc. Cel.*, Livre II):

$$
P-\begin{cases}
c=\dfrac{xdy-ydx}{dt},\ c'=\dfrac{xdz-zdx}{dt},\ c''=\dfrac{ydz-zdy}{dt} \\
0=f\ +x\left[\dfrac{k^2}{r}-\dfrac{dy^2+dz^2}{dt^2}\right]+\dfrac{ydydx}{dt^2}+\dfrac{zdzdx}{dt^2} \\
0=f'\ +y\left[\dfrac{k^2}{r}-\dfrac{dx^2+dz^2}{dt^2}\right]+\dfrac{xdxdy}{dt^2}+\dfrac{zdzdy}{dt^2} \\
0=f''+z\left[\dfrac{k^2}{r}-\dfrac{dx^2+dy^2}{dt^2}\right]+\dfrac{xdxdz}{dt^2}+\dfrac{ydydz}{dt^2} \\
0=\dfrac{k^2}{a}-\dfrac{zk^2}{r}+\dfrac{dx^2+dy^2+dz^2}{dt^2}
\end{cases}
$$

where c, c', c'', f, f', f'', and a are arbitrary constant quantities. The elements of the orbit are the arbitrary constant quantities of its motion, they are consequently functions of the above arbitrary constant quantities. They may be derived by means of the following formulae:

$$\tan \Omega=\frac{c''}{c'}$$

$$\tan\ i=\frac{\sqrt{c'^2+c''^2}}{c}$$

$$\tan\ I=\frac{f'}{f}$$

$$h^2=r^2\left(\frac{dx^2+dy^2+dz^2}{dt^2}\right)-\frac{r^2dr^2}{dt^2}=c'^2+c''^2+c^2$$

$$e=\sqrt{1-\frac{h^2}{k^2a}}$$

where $I\equiv$ longitude of the projection of the perihelion on the fundamental plane.

In the special problem of finding the relative orbit of the comet about Jupiter, we note that k^2 now becomes the acceleration at unit's distance due to the force exerted by the mass of Jupiter: and as the forces of gravitation are directly proportional to the mass of the attracting body, this becomes,

$$k'^2=m'k^2$$

where k^2 is the acceleration due to the sun, namely

$$(8.23558144)^2$$

and m' is the ratio of the mass of Jupiter to that of the sun, namely

$$\frac{1}{1047.879}$$

whence

$$\log k'^2 = 3.4508517$$

Substituting in formulae (P) the numerical values of the relative coördinates and of their differential coefficients as before derived for October 26.0, we find,

$$\begin{array}{llll} \log c & = 4.8989042 & \log f = & 2.2265767\,n \\ \text{"}\quad c' & = 4.6811876\,n & \text{"}\quad f = & 3.4147952\,n \\ \text{"}\quad c'' & = 5.2903906\,n & \text{"}\quad a = & 8.9374383\,n \end{array}$$

From these are deduced,

$$\begin{array}{rl} \Omega = & 256^\circ\ 11'\ 2''.0 \\ i = & 68\ \ 29\ \ 0.85 \\ I = & 266\ \ 17\ \ 26.37 \\ \log a = & 8.9374383\,n \\ \text{"}\quad e = & 0.0041062 \\ \text{"}\quad r = & 9.4580166 \end{array}$$

and the relative orbit is therefore hyperbolic. To find T, the time of perijovian passage we have,

$$r = a\left[\frac{e}{\cos F} - 1\right]$$

whence as r, a and e are now known, we find

$$\cos F = 9.3690526$$
$$F = 76^\circ\ 28'\ 21''.0$$

Again,

$$\frac{\lambda k}{a^{\frac{3}{2}}}(t - T) = N = e\lambda \tan F - \log \tan\left(45^\circ + \frac{1}{2}F\right)$$

where

$$\log \lambda = 9.6377843$$

hence,

$$\frac{\lambda k}{a^{\frac{3}{2}}} \equiv \nu = 7.9570527$$

From the last member of the above equation is found,

$$N = 0.8963864$$

whence,

$$t - T = 98.95612 \text{ days}$$

and consequently the peri-jovian passage took place,

1886, July 19.04388 Gr. M. T.

To find from I the longitude of the peri-jove we have, putting

$$I - \Omega \equiv \omega' \text{ and } \pi - \Omega \equiv \omega$$

$$\cot \omega = \cot \omega' \cos i$$

whence as

$$\omega' = 10^\circ \ \ 6' \ 24''.57$$

we have

$$\omega = 25^\circ \ 55' \ 12''.00$$

Collecting these results we have for the complete hyperbolic elements of the comet about Jupiter on October 26.0, 1886, Gr. M. T., the following:

$$\begin{aligned} \pi &= 282^\circ \ \ 6' \ 14''.0 \\ \Omega &= 256 \ \ 11 \ \ \ 2.0 \\ i &= \ \ 68 \ \ 29 \ \ \ 0.8 \\ \omega &= \ \ 25 \ \ 55 \ \ 12.0 \\ \log a &= 8.9374383\, n \\ \text{“} \ \ e &= 0.0041062 \\ \text{“} \ \ \nu &= 7.9570527 \\ T &= 1886, \text{ July } 19.04388 \text{ Gr. M. T.} \end{aligned}$$

THIRD. Solar Perturbations.

To compute the perturbations due to the action of the sun on the comet whilst the latter was traversing the above hyperbolic orbit about Jupiter, I made use of the ordinary formula for the perturbations of the rectangular coördinates as given by Watson in the eighth chapter of his Theoretical Astronomy. The equations there derived for the second differential coefficients of the perturbations, with reference only to the first power of the disturbing force, are as follows:

$$\frac{d^2\delta x}{dt^2} = m'k^2\left[\frac{x'-x_0}{\rho^3} - \frac{x'}{r'^3}\right] + \frac{k^2(1+m)}{r_0^3}\left[3\,\frac{x_0}{r_0}\,\delta r - \delta x\right]$$

$$\frac{d^2\delta y}{dt^2} = m'k^2\left[\frac{y'-y_0}{\rho^3} - \frac{y'}{r'^3}\right] + \frac{k^2(1+m)}{r_0^3}\left[3\,\frac{y_0}{r_0}\,\delta r - \delta y\right]$$

$$\frac{d^2\delta z}{dt^2} = m'k^2\left[\frac{z'-z_0}{\rho^3} - \frac{z'}{r'^3}\right] + \frac{k^2(1+m)}{r_0^3}\left[3\,\frac{z_0}{r_0}\,\delta r - \delta z\right]$$

and

$$\delta r = \frac{x_0}{r_0}\,\delta x + \frac{y_0}{r_0}\,\delta y + \frac{z_0}{r_0}\,\delta z$$

where, m', x', y' and z' are respectively the mass and the coördinates of the disturbing body.

In the problem now under consideration k^2 has the value already derived for the Jovian system, and m' is the ratio of the mass of the sun as disturbing body, to that of Jupiter as central body, or is the reciprocal of the mass of Jupiter as usually given. Whence we have,

$$m' = \frac{1}{m_j}, \qquad k^2 = m_j k_s^2, \qquad \text{hence} \qquad m'k^2 = k_s^2$$

$$k^2\,(1+m) = m_j k_s^2$$

Substituting numerical values these become,

$$\log\, m'k^2 = 6.47116$$

$$\log\, k^2\,(1+m) = 3.45085$$

And for an interval ω the above equations for the differential coefficients become when expressed in units of the seventh decimal place:

$$\omega^2\,\frac{d^2\delta x}{dt^2} = \omega^2\,[3.47116]\left[\frac{x'-x_0}{\rho^3} - \frac{x'}{r'^3}\right] + \omega^2\,\frac{[3.45085]}{r_0^3}\left(3\,\frac{x_0}{r_0}\,\delta r - \delta x\right)$$

$$\omega^2\,\frac{d^2\delta y}{dt^2} = \omega^2\,[3.47116]\left[\frac{y'-y_0}{\rho^3} - \frac{y'}{r'^3}\right] + \omega^2\,\frac{[3.45085]}{r_0^3}\left(3\,\frac{y_0}{r_0}\,\delta r - \delta y\right)$$

$$\omega^2\,\frac{d^2\delta z}{dt^2} = \omega^2\,[3.47116]\left[\frac{z'-z_0}{\rho^3} - \frac{z'}{r'^3}\right] + \omega^2\,\frac{[3.45085]}{r_0^3}\left(3\,\frac{z_0}{r_0}\,\delta r - \delta z\right)$$

The first term of the second members of these equations can be computed directly for all the required dates. The second terms, however, can only be obtained by a process of approximation as they contain the quantities δr and δx or δy or δz. We first derive approximate values of $\omega^2\,\frac{d^2\delta x}{dt^2}$, $\omega^2\,\frac{d^2\delta y}{dt^2}$ and $\omega^2\,\frac{d^2\delta z}{dt^2}$ by neglecting the second terms. These are

integrated by the usual formulae for integration and with the resulting approximate values of δx, δy and δz we complete the values of the differential coefficients and integrate anew. In this way the perturbations may be carried back from date to date. In some cases, however, two and even three or more approximations had to be made.

From October 26th to August 17th, a ten day interval was used. But on the latter date the indirect terms in the above equations became too large for convenience or for accuracy. It was necessary, therefore, to apply the perturbations to the elements, and thus to obtain a new set of osculating elements. This transformation is effected by means of the values of the perturbations of the coördinates, with the corresponding values of the variations of the velocities,

$$\frac{dx}{dt}, \qquad \frac{dy}{dt}, \qquad \frac{dz}{dt}$$

The variations of these quantities, or

$$\frac{d\delta x}{dt}, \qquad \frac{d\delta y}{dt}, \qquad \frac{d\delta z}{dt}$$

are obtained by a single integration from the second differential coefficients already obtained. Then we have for the date required

$$x = x_0 + \delta x, \qquad \frac{dx}{dt} = \frac{dx_0}{dt} + \frac{d\delta x}{dt}$$

$$y = y_0 + \delta y, \qquad \frac{dy}{dt} = \frac{dy_0}{dt} + \frac{d\delta y}{dt}$$

$$z = z_0 + \delta z, \qquad \frac{dz}{dt} = \frac{dz_0}{dt} + \frac{d\delta z}{dt}$$

From these values the new osculating elements may be computed by equations (P) as before.

To find the values of $\frac{dx_0}{dt}, \frac{dy_0}{dt}, \frac{dz_0}{dt}$ from the hyperbolic elements, we have

$$N = \lambda N_0 \qquad \text{and} \qquad N_0 = e \tan F - \log_e \tan\left(45^\circ + \frac{1}{2} F\right)$$

Differentiating with respect to the time, but keeping the elements constant, we get,

$$\frac{dN}{dt} = \lambda \frac{dN_0}{dt}, \qquad \frac{dN_0}{dt} = \left(\frac{e}{\cos F} - 1\right) \frac{dF}{dt} \frac{1}{\cos F}$$

But from the theory of hyperbolic motion,

$$r = a\left(\frac{e}{\cos F} - 1\right)$$

Substituting in the above,

$$\frac{dF}{dt} \cdot \frac{1}{\sin F} = \frac{a}{r \tan F} \cdot \frac{dN_0}{dt}$$

As the elements are constant we have,

$$\frac{dN}{dt} = \nu$$

Taking the logarithms of both members of the equation,

$$\tan\frac{1}{2}F = \tan\frac{1}{2}v \tan\frac{1}{2}\psi$$

and differentiating,

$$\frac{dv}{dt} = \frac{\sin v}{\sin F} \cdot \frac{dF}{dt}$$

And from the above value of r, we have by differentiating,

$$\frac{dv}{dt} = \frac{ae \tan^2 F}{\sin F} \cdot \frac{dF}{dt}$$

Again, $u = v + \pi - \Omega$, whence

$$\frac{du}{dt} = \frac{dv}{dt}$$

The values of x, y and z and $D_t x$, $D_t y$ and $D_t z$ may now be obtained from the hyperbolic elements in a manner entirely similar to that by which the corresponding values were obtained from the elliptic elements on October 26th. That is,

$$x = r \sin a \sin (A + u)$$
$$y = r \sin b \sin (B + u)$$
$$z = r \sin c \sin (C + u)$$

and

$$D_t x = \frac{x}{r} D_t r + r \sin a \cos (A + u)\, D_t u$$

$$D_t y = \frac{y}{r} D_t r + r \sin b \cos (B + u)\, D_t u$$

$$D_t z = \frac{z}{r} D_t r + r \sin i \cos u\, D_t u$$

From the osculating hyperbolic elements of October 26th, we thus find for August 17th:

$$F = 64^\circ\ 0'.16 \qquad \log\ r = 9.05169$$
$$v = 167\ \ 26.54 \qquad ''\ D_t u = 7.23045$$
$$u = 193\ \ 21.74 \qquad ''\ D_t r = 7.45804$$

and

$$\log x_0 = 8.22788 \qquad \log D_t x_0 = 6.54880$$
$$''\ y_0 = 9.03623 \qquad ''\ D_t y_0 = 7.43838$$
$$''\ z_0 = 8.38413\ n \qquad ''\ D_t z_0 = 6.89795\ n$$

From the table of perturbations, computed as before explained, we have for August 17th by integrating:

$$\log \delta x = 6.99648 \qquad \log D_t \delta x = 5.35031\ n$$
$$''\ \delta y = 6.86291\ n \qquad ''\ D_t \delta y = 5.29714$$
$$''\ \delta z = 6.39262 \qquad ''\ D_t \delta z = 4.79810\ n$$

Applying these to the values of x_0, y_0, z_0, etc., we have for the corrected values of the rectangular coördinates and their differential coefficients for August 17.0:

$$\log x = 8.25265 \qquad \log D_t x = 6.52040$$
$$''\ y = 9.03331 \qquad ''\ D_t y = 7.44151$$
$$''\ z = 8.37968\ n \qquad ''\ D_t z = 6.90139\ n$$

From these were deduced the new elements, as follows:

$$\pi = 281^\circ\ 41'.49$$
$$\Omega = 252\ \ 18.49$$
$$i = 56\ \ 26.41$$
$$\omega = 29\ \ 23.00$$
$$\log a = 8.92681\ n$$
$$''\ e = 0.00549$$
$$''\ \nu = 7.97300$$
$$T = \text{July } 19.3273 \text{ Gr. M. T.}$$

With these as osculating elements the perturbations were continued until July 20th, using a four day interval. As the peri-jove is approached

r_0^3 becomes a very small quantity, and hence the coefficient of the indirect term of the differential coefficient becomes very large; and the term itself more and more difficult to approximate to in value. It was necessary, therefore, to again apply the perturbations and to deduce new elements for July 24th. The perturbations were then continued until July 4th, when they were again applied and new osculating elements obtained. With these elements of July 4th, the perturbations were carried back, using a period of ten days, to March 26th, on which day the comet passed out of the sphere of Jupiter's activity. These various sets of elements are given below, and the perturbations are given in the tables at the end of the thesis:

HYPERBOLIC ELEMENTS.

	Oct. 26.0	Aug. 17.0	July 24.0
$\pi =$	282° 6′ 14″.3	281° 41′.49	281° 44′.50
$\Omega =$	256 11 20.0	252 18.49	252 11.36
$i =$	68 29 0.8	56 26.41	56 1.63
$\omega =$	25 55 12.3	29 23.00	29 33.14
$\log a =$	8.9374383 n	8.92681 n	8.92450 n
" $e =$	0.0041062	0.00549	0.00558
" $\nu =$	7.9570528	7.97300	7.97646
$T =$	July 19.0439	19.3273	19.3306

	July 4.0	March 26.0
$\pi =$	281° 42′.85	283° 48′.33
$\Omega =$	252 12.28	255 8.06
$i =$	56 2.00	58 55.36
$\omega =$	29 30.57	28 40.27
$\log a =$	8.92423 n	8.94800 n
" $e =$	0.00558	0.00479
" $\nu =$	7.97686	7.94121
$T =$	19.3296	20.1238

From these we see that the solar perturbations produce quite marked changes in the relative hyperbolic orbit about Jupiter. For nearly a month

of the closest approach, however, these perturbations are very small, and but very slightly affect the character of the resulting orbit. During this time also the computation is very difficult, owing to the very rapid motion of the comet in its orbit, and to its very small distance from the planet. A very good approximation to the effect of the solar perturbations can be obtained by neglecting entirely the action of the sun as a disturbing body for perhaps two weeks before and after the comet passed its peri-jove.

The remarkable character of the appulse is seen by the elements of July 24.0. The comet passed the centre of Jupiter on 1886, July 19.33, at no greater distance than two and a third radii of that planet. It must then have passed the surface of the planet at a distance of only one and a third radii; that is the centre of the comet was only 57,650 miles from the surface of the planet. It is not at all improbable that parts of the diffused mass of the comet may have swept over the surface of the planet itself; and this, together with the unequal attractions of the planet's satellites upon the different parts of the comet may have tended to disruption, and caused the separation actually observed.

Two diagrams are given, showing the character of the orbit about Jupiter. The first of these, *A*, represents the projection of the relative hyperbolic orbit about Jupiter upon the plane of the ecliptic. The figure also contains the orbits of the two outer satellites of Jupiter. Figure *B* shows the true paths of Jupiter and the comet projected upon the plane of the ecliptic. The slightly curved line represents that portion of Jupiter's orbit traversed between March and October 1886; on this the positions of the planet for various intermediate dates are shown. The full curved line represents the actual orbit of the comet during the same period, and on it are shown the various positions of the comet for the same dates. On March 26th the comet is shown just entering the sphere of Jupiter's activity, at a point outside the orbit of that planet; as the two bodies proceed the comet approaches closer and closer to Jupiter until July 20th, when it attains its nearest approach. At this time it crosses the planet's orbit and henceforth continues its course within the orbit of Jupiter, and is shown leaving the sphere of activity on October 26th. Near the point of closest approach is found a point of inflection in the comet's orbit.

PART THIRD.

TRANSFORMATION FROM JUPITER TO THE SUN AS CENTRAL BODY.

The date chosen for this transformation was March 26.0. at which time the comet was distant from Jupiter,

$$\log \rho = 9.5128642$$

The process of the transformation is entirely similar to that already explained when we passed from the sun to Jupiter on October 26.0. From the hyperbolic elements about Jupiter are derived the rectangular coördinates and their differential coefficients referred to Jupiter as a centre. From these and from the corresponding quantities for Jupiter referred to the sun as centre are obtained the coördinates of the comet and their differential coefficients referred to the sun. The elements of the comet's orbit about the sun are then derived by means of equations *P*.

For March 26.0 the relative coördinates of the comet and their differential coefficients referred to Jupiter as a centre are:

$\log x = 8.5792316\,n$	$\log D_t x = 6.3416455$
" $y = 9.4310521$	" $D_t y = 7.2748372\,n$
" $z = 9.2448044\,n$	" $D_t z = 7.0621004$

From the *British Nautical Almanac* we find, in the manner already explained (page 17), for the position of Jupiter on March 26.0,

$$\begin{aligned}
\lambda' &= \ 181^\circ\ 30'\ 10''.1 \\
\beta' &= \ \ \ \ \ 1\ \ 17\ \ 51.1 \\
\log r' &= \ 0.7365048 \\
D_t\lambda' &= +\ \ 0^\circ\ \ 4'\ \ 32''.3 \\
D_t\beta' &= +\ \ 0\ \ \ 0\ \ \ 0.85 \\
\log D_t r &= \ 5.8621048
\end{aligned}$$

Reducing $D_t\lambda$ and $D_t\beta$ to radians, and substituting as before, we find, for the rectangular coördinates of Jupiter referred to the sun and their differential coefficients,

$$\begin{aligned} \log\ x' &= 0.7362440\,n \\ \text{"}\quad y' &= 9.1551237\,n \\ \text{"}\quad z' &= 9.0914617 \\ \text{"}\quad D_tx' &= 6.0661227 \\ \text{"}\quad D_ty' &= 7.8580167\,n \\ \text{"}\quad D_tz' &= 5.3821473 \end{aligned}$$

Combining these with the above given relative coördinates of the comet referred to Jupiter, we find, for the rectangular coördinates and their differential coefficients of the comet referred to the sun as centre,

$$\begin{aligned} \log\ x &= 0.7392581\,n \\ \text{"}\quad y &= 9.1033802 \\ \text{"}\quad z &= 8.7182655\,n \\ \text{"}\quad D_tx &= 6.5264067 \\ \text{"}\quad D_ty &= 7.9587691\,n \\ \text{"}\quad D_tz &= 7.0710816 \end{aligned}$$

Substituting these values in formulae (P) and remembering that k^2 now has its usual value, *i. e.*, the acceleration at unit's distance due to the force exerted by the sun, we have,

$$\begin{aligned} \log\ c &= 8.6976559 & \log\ f &= 6.1977764\,n \\ \text{"}\quad c' &= 7.8091574\,n & \text{"}\quad f' &= 5.3797986\,n \\ \text{"}\quad c'' &= 6.5131308\,n & \text{"}\quad a &= 1.0979460 \end{aligned}$$

From these are deduced:

$$\begin{aligned} \Omega &= 182^\circ\ 53'\ 43''.86 \\ i &= 7\ \ 22\ \ 30.55 \\ I &= 188\ \ 38\ \ 46.7 \\ \log\ \dot{a} &= 1.0979460 \\ \text{"}\quad e &= 9.7513910 \\ \text{"}\quad r &= 0.7393938 \\ \text{"}\quad \mu &= 1.9030876 \end{aligned}$$

The resulting orbit is therefore elliptic. To find the time of perihelion passage we have from the theory of elliptic motion,

$$\cos E = \frac{a - r}{ae}$$

and

$$M = E - e'' \sin E$$

Whence we have,

$$E = -4^\circ\ 58'\ 2''.5$$
$$M = -2\ \ 10\ \ 6.8$$
$$t - T = -97.5855 \text{ days.}$$

Finding π from I as before, we have for the elliptic elements of the comet about the sun on March 26.0, the following:

$$\left.\begin{aligned} \pi &= 188^\circ\ 41'\ 38''.2 \\ \Omega &= 182\ \ 53\ \ 43.8 \\ i &= \ \ 7\ \ 22\ \ 30.6 \\ \omega &= \ \ 5\ \ 47\ \ 54.4 \end{aligned}\right\} 1890.0$$

$$\log a = 1.0979460$$
$$\text{“} \quad e = 9.7513910$$
$$\text{“} \quad \mu = 1.9030876$$
$$T = 1886 \text{ July } 1.5855$$

PART FOURTH.

While the results here derived confirm Mr. Chandler's conclusions as to the identity of the two comets, yet they differ radically from his in several very important particulars. The comet is found to have approached Jupiter much closer than he suspected, and the resulting changes in the orbit are much more radical. Before the appulse in 1886, the orbit is found to be much larger than his work seemed to indicate, the period being over forty years instead of only twenty-seven. The position of the orbit in space, however, is but slightly different from that deduced by him; the inclination and the line of nodes are practically unaltered, the longitude of the perihelion is changed but a few degrees from the value assigned in his paper.

The radical character of the changes in the comet's orbit, caused by the appulse, are shown in Plate II. Here are represented the orbits of the earth, Jupiter, and Saturn, together with those of the comet before and after its approach to Jupiter in 1886. The present orbit of the comet is the small full ellipse, with its perihelion beyond the earth's orbit, and its aphelion just outside the orbit of Jupiter in longitude 180° approximately. On this curve the positions of the comet are shown at the time of discovery on July 6th, 1889, at perihelion passage, and at the date of the last observation in December, 1890. The large heavy ellipse reaching far beyond the orbit of Saturn is the orbit of the comet before its disturbance in 1886. These two paths are tangent to each other near the point of closest approach in heliocentric longitude 180°. The disturbance completely reversed the orbit, the perihelion before that date coincides almost exactly with the present aphelion. The large old orbit was one that the comet traversed in about forty years; the present, one of only seven years. On this large curve is shown the position of the comet in March, 1886, as it was approaching conjunction with Jupiter. This orbit

shows that the comet passed very close to the orbit of Saturn in heliocentric longitude 90° and 290°, and that at either one of these points it is possible to have a close approach of the two bodies. The plate also shows the original orbit of Lexell's comet as a small dotted ellipse of large eccentricity with its aphelion coinciding very nearly with the aphelion of the present orbit of Comet V.

The point upon which Mr. Chandler laid so much stress, that the period of the comet before 1886 was approximately 26.5 years, seems to be overthrown. Assuming the substantial correctness of his period he showed that four periods of the comet (107.8 yrs.) are nearly equal to nine of Jupiter's (106.9 yrs.), and that the two bodies would have had, therefore, another close approach in 1779. His conclusions as to the identity of Comet V 1889 and Lexell 1770, depend entirely upon the assumption of four revolutions of the comet during the hundred and seven years between the appulse. For, according to him, three revolutions during this period would make one period of the comet very nearly equal to three of Jupiter's (35.6 yrs), and cause close approaches of the two bodies in 1850 and again in 1875. Such approaches and the consequent enormous changes in the orbit render all attempts to determine the character of the orbit in 1779 utterly fruitless; and consequently with the supposition of three revolutions of equal periods between 1886 and 1779, we cannot hope to establish the identity of these two remarkable bodies.

My work, which gives a period of about forty-four (44) years for the comet before 1886, seems to indicate only three revolutions in the 107 years to be accounted for in order to establish identity, but revolutions of unequal periods owing to large perturbations by Saturn. The comet, according to my results, was at its shortest distance from Saturn's orbit about 1846, in heliocentric longitude 295°, and Saturn was at the same point in its orbit about 1844.7. While I have not had time to determine accurately the character of this approach, yet it seems probable that the resulting changes in the comet's orbit were large and sufficient to have changed the period by a few years. I made a very hurried and rough approximation to the effect of the perturbations of Jupiter for a few months before the appulse in 1886, and also as to the character of the

perturbations by Saturn in 1846, and found that the period was shortened considerably. This would seem to indicate a little more than two complete revolutions of about 34 years each between 1779 and 1846, and a nearly complete revolution of about 44 years from that date to 1886. Mr. Schulhof arrived at practically the same conclusions by an entirely different method.

His first paper appeared in the *Bulletin Astronomique*, November, 1889. He there discussed the possibility of the identity of several pairs of periodic comets, by means of a criterion formulated by M. Tisserand. A quantity, n, is derived, which, if two comets seen at different apparitions are identical, must be the same. The value of n is given by the formula,

$$n = \frac{1}{a} + \frac{2\sqrt{A}}{R^2} \sqrt{p} \cos i$$

where, a, p and i are respectively the semi-major axis, parameter and inclination of the orbit, and A and R the semi-major axis and radius vector of any disturbing planet at the point of nearest approach. For a single comet the perturbations by a single planet can produce only a small variation in the value of n. Mr. Schulhof finds (*Bulletin Astronomique*, December, 1889), using Le Verrier's elements of Lexell's comet and Chandler's of Comet V, that this criterion can only be satisfied for these two comets upon the supposition of a strong perturbation by Saturn. Assuming the identity of the two comets, he deduces by means of the criterion the most probable orbit of the comet between 1779 and 1886, and finds its period to have been about 32 years from 1779 to 1849, at which time the perturbations of Saturn increased its period to about 42 years.

This agrees so strikingly with the results of my direct computation of these intermediate orbits, that there can be, I think, no doubt as to the identity of these two comets.

Charles Lane Poor, the third son of Edward Eri and Mary (Lane) Poor, was born at Hackensack, New Jersey, on January 18th, 1866. After studying in private and public schools he entered the Introductory Department of the College of the City of New York in the fall of 1880 and was graduated from that Institution with the degree of Bachelor of Sciences in June, 1886. The following October he was appointed a Tutor in that college. This position he held for two years, teaching Descriptive Geometry and Mathematics.

In October, 1888, he resigned the above position to enter the Johns Hopkins University, where he became a student of Astronomy and Physics. In January, 1889, he was appointed University Scholar in Astronomy, and, at the following Commencement, Fellow in Astronomy.

The degree of Master of Sciences was conferred on him by the College of the City of New York in June, 1889.

During his graduate study he has attended courses given by Professors Simon Newcomb, Henry A. Rowland and Doctors Craig and Kimball, all of the Johns Hopkins University.

TABLES.

PERTURBATIONS BY JUPITER.

	$\omega \frac{d\phi}{dt}$	$'f$	$\omega \frac{dL}{dt}$	$'f$
		+ 9376.40		+ 7157.20
1886, Oct. 26,	− 1306.00		− 892.70	
		+ 8070.40		+ 6264.50
Nov. 5,	− 1116.80		− 783.20	
		+ 6953.60		+ 5481.30
15,	− 964.60		− 692.70	
		+ 5989.00		+ 4788.60
25,	− 858.60		− 620.30	
		+ 5130.40		+ 4168.30
Dec. 5,	− 758.50		− 562.20	
		+ 4371.90		+ 3606.10
15,	− 672.80		− 512.10	
		+ 3699.10		+ 3094.00
25,	− 591.80		− 457.80	
		+ 3107.30		+ 2636.20
1887, Jan. 4,	− 545.60		− 432.70	
		+ 2561.70		+ 2203.50
14,	− 497.00		− 402.10	
		+ 2064.70		+ 1801.40
24,	− 452.90		− 377.10	
		+ 1611.80		+ 1424.30
Feb. 3,	− 414.50		− 351.40	
		+ 1197.30		+ 1072.90
13,	− 380.00		− 330.20	
		+ 817.30		+ 742.70
23,	− 348.60		− 310.30	
		+ 468.70		+ 432.40
Mch. 5,	− 319.30		− 292.90	
		+ 149.40		+ 139.50
15,	− 294.90	+ (2.00)	− 276.50	+ (1.25)
25,	− 270.80		− 261.90	

	$\omega \frac{d\phi}{dt}$	$'f$	$\omega \frac{dL}{dt}$	$'f$
		+ 4942.59		+ 6302.13
1887, Mch. 15,	− 1032.82		− 958.26	
		+ 3909.77		+ 5343.87
Apr. 24,	− 795.59		− 811.32	
		+ 3114.18		+ 4532.45
June 3,	− 622.67		− 698.62	
		+ 2491.51		+ 3833.83
July 13,	− 494.96		− 610.55	
		+ 1996.55		+ 3223.28
Aug. 22,	− 394.42		− 533.46	
		+ 1602.13		+ 2689.82
Oct. 1,	− 317.39		− 472.09	
		+ 1284.74		+ 2217.73
Nov. 10,	− 254.26		− 412.43	
		+ 1030.48		+ 1805.30
Dec. 20,	− 204.29		− 360.06	
		+ 826.19		+ 1445.24
1888, Jan. 29,	− 164.14		− 311.22	
		+ 662.05		+ 1134.02
Mch. 9,	− 131.39		− 265.05	
		+ 530.66		+ 868.97
Apr. 18,	− 105.26		− 221.79	
		+ 425.40		+ 647.18
May 28,	− 84.68		− 181.68	
		+ 340.72		+ 465.50
July 7,	− 68.58		− 144.82	
		+ 272.14		+ 320.68
Aug. 16,	− 56.21		− 111.89	
		+ 215.93		+ 208.79
Sept. 25,	− 46.75		− 83.10	
		+ 169.18		+ 125.69
Nov. 4,	− 39.56		− 58.81	
		+ 129.62		+ 66.88
Dec. 14,	− 35.38		− 39.70	
		+ 94.24		+ 27.18
1889, Jan. 23,	− 28.88		− 23.61	
		+ 65.36		+ 13.57
Mch. 4,	− 24.43		− 12.12	
		+ 40.93		+ 1.45
Apr. 13,	− 20.00		− 4.12	
		+ 20.93		− 2.67
May 23,	− 15.33		+ 0.97	
		+ 5.60		− 1.70
July 2,	− 10.36	+ (0.42)	+ 3.73	+ (0.16)
Aug. 11,	− 5.33		+ 4.79	
Sept. 20,	− 0.69		+ 4.72	

	$\omega^2 \frac{d^2\mu}{dt}$	$'f$	$''f$	$\omega \frac{di}{dt}$	$'f$
		+ 180.68			+ 3470.25
1886, Oct. 26,	− 25.50		− 879.92	− 483.10	
		+ 155.18			+ 2987.15
Nov. 5,	− 21.71		− 724.74	− 417.30	
		+ 133.47			+ 2569.85
15,	− 18.67		− 591.27	− 363.65	
		+ 114.80			+ 2206.20
25,	− 16.75		− 476.47	− 315.10	
		+ 98.05			+ 1891.10
Dec. 5,	− 14.72		− 378.42	− 280.70	
		+ 83.33			+ 1610.40
15,	− 13.00		− 295.09	− 239.80	
		+ 70.33			+ 1370.60
25,	− 11.46		− 224.76	− 217.30	
		+ 58.87			+ 1153.30
1887, Jan. 4,	− 10.51		− 165.89	− 201.80	
		+ 48.36			+ 951.50
14,	− 9.55		− 117.53	− 182.50	
		+ 38.81			+ 769.00
24,	− 8.65		− 78.72	− 167.50	
		+ 30.16			+ 601.50
Feb. 3,	− 7.84		− 48.56	− 154.00	
		+ 22.32			+ 447.50
13,	− 7.10		− 26.24	− 141.80	
		+ 15.22			+ 305.70
23,	− 6.51		− 11.02	− 129.90	
		+ 8.71			+ 175.80
Mch. 5,	− 5.95		− 2.31	− 119.50	
		+ 2.76			+ 56.30
15,	− 5.45	+ (0.04)	+ (0.45)	− 111.30	+ (0.70)
25,	− 5.01			− 102.30	

			+ 624.51		
		+ 178.06			+ 1731.89
1887, Mch. 15,	− 78.49		+ 802.57	− 363.58	
		+ 99.57			+ 1368.31
Apr. 24,	− 58.33		+ 902.14	− 286.85	
		+ 41.24			+ 1081.46
June 3,	− 43.42		+ 943.38	− 229.27	
		− 2.18			+ 852.19
July 13,	− 33.26		+ 941.20	− 185.65	
		− 35.44			+ 666.54

	$\omega^2 \frac{d^2\mu}{dt}$	$'f$	$''f$	$\omega \frac{di}{dt}$	$'f$
1887, Aug. 22,	− 23.12		+ 905.76	− 150.07	
		− 58.56			+ 516.47
Oct. 1,	− 15.77		+ 847.20	− 122.03	
		− 74.33			+ 394.44
Nov. 10,	− 9.87		+ 772.87	− 97.79	
		− 84.20			+ 296.65
Dec. 20,	− 4.78		+ 688.67	− 78.17	
		− 88.98			+ 218.48
1888, Jan. 29,	− 0.69		+ 599.69	− 61.68	
		− 89.67			+ 156.80
Mch. 9,	+ 2.72		+ 510.02	− 47.65	
		− 86.95			+ 109.15
Apr. 18,	+ 5.41		+ 423.07	− 35.97	
		− 81.54			+ 73.18
May 28,	+ 7.42		+ 341.53	− 26.41	
		− 74.12			+ 46.77
July 7,	+ 8.81		+ 267.41	− 18.68	
		− 65.31			+ 28.09
Aug. 16,	+ 9.62		+ 202.10	− 12.63	
		− 55.69			+ 15.46
Sept. 25,	+ 9.88		+ 146.41	− 8.05	
		− 45.81			+ 7.41
Nov. 4,	+ 9.69		+ 100.60	− 4.72	
		− 36.12			+ 2.69
Dec. 14,	+ 9.19		+ 644.48	− 2.49	
		− 26.93			+ 0.20
1889, Jan. 23,	+ 8.21		+ 37.55	− 0.97	
		− 18.72			− 0.77
Mch. 4,	+ 7.08		+ 18.83	− 0.11	
		− 11.64			− 0.88
Apr. 13,	+ 5.76		+ 7.19	+ 0.29	
		− 5.88			− 0.59
May 23,	+ 4.33		+ 1.31	+ 0.41	
		− 1.55			− 0.18
July 2,	+ 2.87	− (0.12)	− (0.24)	+ 0.35	− (0.01)
Aug. 11,	+ 1.44			+ 0.20	
Sept. 20,	+ 0.19			+ 0.03	

	$\omega \frac{d\Omega}{dt}$	$'f$	$\omega \frac{d\pi}{dt}$	$'f$
		− 719.60		+ 2445.70
1886, Oct. 26,	+ 280.10		− 230.40	
		− 439.50		+ 2215.30
Nov. 5,	+ 209.60		− 214.20	
		− 229.90		+ 2001.10
15,	+ 154.40		− 205.40	
		− 75.50		+ 1795.70
25,	+ 121.70		− 195.10	
		+ 46.20		+ 1600.60
Dec. 5,	+ 84.90		− 188.40	
		+ 131.10		+ 1412.20
15,	+ 52.40		− 176.80	
		+ 183.50		+ 1235.40
25,	+ 31.20		− 168.70	
		+ 214.70		+ 1066.70
1887, Jan. 4,	+ 11.20		− 163.70	
		+ 225.90		+ 903.00
14,	− 3.20		− 159.70	
		+ 222.70		+ 743.30
24,	− 16.80		− 151.60	
		+ 205.90		+ 591.70
Feb. 3,	− 28.40		− 144.00	
		+ 177.50		+ 447.70
13,	− 39.40		− 136.30	
		+ 138.10		+ 311.40
23,	− 49.70		− 130.20	
		+ 88.40		+ 181 20
Mch. 5,	− 56.00		− 122.70	
		+ 32.40		+ 58.50
15,	− 66.00	− (0.65)	− 116.20	+ (0.40)
25,	− 71.90		− 111.20	

		+ 4737.22		+ 1579.45
1887, Mch. 15,	− 292.76		− 416.80	
		+ 4444.46		+ 1162.65
Apr. 24,	− 356.60		− 363.68	
		+ 4087.86		+ 798.97
June 3,	− 388.12		− 311.95	
		+ 3699.74		+ 487.02
July 13,	− 402.92		− 262.64	
		+ 3296.82		+ 223.38
Aug. 22,	− 401.88		− 217.50	
		+ 2894.94		+ 5.88

	$\omega \frac{d\Omega}{dt}$	$'f$	$\omega \frac{d\pi}{dt}$	$'f$
1887, Oct. 1,	− 393.40		− 171.96	
		+ 2501.54		− 166.08
Nov. 10,	− 373.27		− 130.56	
		+ 2128.27		− 296.64
Dec. 20,	− 349.56		− 91.43	
		+ 1778.71		− 388.07
1888, Jan. 29,	− 320.00		− 56.31	
		+ 1458.71		− 444.38
Mch. 9,	− 287.81		− 23.97	
		+ 1170.90		− 468.35
Apr. 18,	− 252.80		+ 3.63	
		+ 918.10		− 464.72
May 28,	− 216.85		+ 26.01	
		+ 701.25		− 438.71
July 7,	− 181.00		+ 42.99	
		+ 520.25		− 395.72
Aug. 16,	− 146.94		+ 54.39	
		+ 373.31		− 341.33
Sept. 25,	− 115.54		+ 61.19	
		+ 257.77		− 280.14
Nov. 4,	− 87.75		+ 61.19	
		+ 170.02		− 218.95
Dec. 14,	− 65.47		+ 58.84	
		+ 104.55		− 160.11
1889, Jan. 23,	− 44.70		+ 51.54	
		+ 59.85		− 108.57
Mch. 4,	− 29.44		+ 42.96	
		+ 30.41		− 65.61
Apr. 13,	− 17.96		+ 33.33	
		+ 12.45		− 32.28
May 23,	− 9.84		+ 23.84	
		+ 2.61		− 8.44
July 2,	− 4.53	+ (0.35)	+ 15.72	− (0.58)
Aug. 11,	− 1.47		+ 9.91	
Sept. 20,	− 0.11		+ 7.00	

SOLAR PERTURBATIONS.

	$\omega^2 \frac{d^2\delta x}{dt^2}$	$'f$	$''f$	δx
		+ 359.89		
Mch. 26,	− 126.72		− 332.27	− 342.83
		+ 233.17		
Apr. 5,	− 97.20		− 99.10	− 107.20
		+ 135.97		
15,	− 71.04		+ 36.87	+ 30.95
		+ 64.93		
25,	− 48.84		+ 101.80	+ 97.73
		+ 16.09		
May 5,	− 30.36		+ 117.89	+ 115.36
		− 14.27		
15,	− 14.76		+ 103.62	+ 102.39
		− 29.03		
25,	− 2.64		+ 74.59	+ 74.37
		− 31.67		
June 4,	+ 6.12		+ 42.92	+ 43.43
		− 25.55		
14,	+ 11.22		+ 17.37	+ 18.30
		− 14.30		
24,	+ 10.68		+ 3.07	+ 3.96
		− 3.62		
July 4,	+ 6.56	− (0.34)	− (0.55)	+ 0.00

		+ 15.94		
July 4,	+ 1.38		− 66.63	− 66.51
		+ 17.32		
8,	+ 0.72		− 49.31	− 49.25
		+ 18.04		
12,	− 0.02		− 31.27	− 31.272
		+ 18.02		
16,	− 2.40		− 13.25	− 13.45
		+ 15.62		
20,	− 18.72		+ 2.37	+ 0.81
		− 3.10		
24,	+ 8.79	+ (1.30)	− (0.73)	0.00
		+ 5.70		
28,	+ 12.60		+ 4.97	

	$\omega^2 \frac{d^2\delta x}{dt^2}$	$'f$	$''f$	δx
		− 2.07		
July 24,	− 74.04		+ 287.18	+ 281.01
		− 76.11		
28,	− 0.72		+ 211.07	+ 211.01
		− 76.83		
Aug. 1,	+ 12.84		+ 134.24	+ 135.31
		− 63.99		
5,	+ 17.06		+ 70.25	+ 71.67
		− 46.93		
9,	+ 21.28		+ 23.32	+ 24.09
		− 25.65		
13,	+ 25.50		− 2.33	− 0.21
		− 0.15		
17,	+ 29.72		− 2.48	0.00

		− 2191.60		
Aug. 17,	− 64.20		+ 9924.60	+ 9919.25
		− 2255.80		
27,	+ 136.44		+ 7668.80	+ 7680.17
		− 2119.36		
Sept. 6,	+ 263.16		+ 5549.44	+ 5571.37
		− 1856.20		
16,	+ 314.52		+ 3693.24	+ 3719.45
		− 1541.68		
26,	+ 377.28		+ 2151.56	+ 2183.00
		− 1164.40		
Oct. 6,	+ 401.64		+ 987.16	+ 1020.63
		− 762.76		
16,	+ 492.48		+ 224.40	+ 265.44
		− 270.28		
26,	+ 550.04	+ (4.74)	− (45.88)	

	$\omega^2 \frac{d^2\delta y}{dt^2}$	$'f$	$''f$	δy
		+ 3491.34		
Mch. 26,	− 513.96		− 12754.77	− 12797.60
		+ 2977.38		
Apr. 5,	− 465.72		− 9777.39	− 9816.20
		+ 2511.66		
15,	− 443.28		− 7265.73	− 7302.67
		+ 2068.38		
25,	− 427.36		− 5197.35	− 5232.13
		+ 1641.02		
May 5,	− 360.00		− 3556.33	− 3586.33
		+ 1281.02		
15,	− 332.24		− 2275.31	− 2302.83
		+ 948.78		
25,	− 290.40		− 1326.53	− 1350.73
		+ 658.38		
June 4,	− 247.80		− 668.15	− 688.80
		+ 410.58		
14,	− 200.28		− 257.57	− 274.26
		+ 210.30		
24,	− 154.44		− 47.27	− 60.14
		+ 55.86		
July 4,	− 103.12	+ (4.28)	+ (8.59)	0.00

		+ 8.66		
July 4,	− 14.94		+ 61.10	+ 59.85
		− 6.28		
8,	− 10.44		+ 54.82	+ 53.95
		− 16.72		
12,	− 5.04		+ 38.10	+ 37.68
		− 21.76		
16,	+ 3.96		+ 16.34	+ 16.67
		− 17.80		
20,	+ 19.80		− 1.46	+ 0.19
		+ 2.00		
24,	− 6.53	− (1.26)	+ (0.54)	0.00
		− 4.52		
28,	− 10.44		− 3.98	

	$\omega^2 \frac{d^2\delta y}{dt^2}$	$'f$	$''f$	δy
July 20,				
		+ 199.64		
24,	− 99.24		− 304.40	− 312.67
		+ 100.40		
28,	− 21.84		− 204.00	− 205.82
		+ 78.56		
Aug. 1,	− 16.92		− 125.44	− 126.85
		+ 61.64		
5,	− 18.52		− 63.80	− 65.34
		+ 43.12		
9,	− 20.50		− 20.68	− 22.39
		+ 22.62		
13,	− 22.52		+ 1.94	+ 0.06
		+ 0.10		
17,	− 24.50		+ 2.04	0.00

		+ 2093.48		
Aug. 17,	− 237.72		− 7263.19	− 7293.00
		+ 1855.76		
27,	− 269.40		− 5407.43	− 5429.88
		+ 1586.36		
Sept. 6,	− 251.16		− 3821.07	− 3842.00
		+ 1335.20		
16,	− 262.92		− 2485.87	− 2507.78
		+ 1072.28		
26,	− 280.92		− 1413.59	− 1437.00
		+ 791.36		
Oct. 6,	− 307.44		− 622.23	− 647.85
		+ 483.92		
16,	− 317.64		− 138.31	− 164.78
		+ 166.28		
26,	− 335.60	− (1.52)	+ (27.97)	

	$\omega^2 \frac{d^2\delta z}{dt^2}$	$'f$	$''f$	δz
		− 2309.40		
Mch. 26,	+ 336.24		+ 8580.58	+ 8608.60
		− 1973.16		
Apr. 5,	+ 308.64		+ 6607.42	+ 6633.14
		− 1664.52		
15,	+ 290.52		+ 4942.90	+ 4967.11
		− 1374.00		
25,	+ 267.24		+ 3568.90	+ 3591.17
		− 1106.76		
May 5,	+ 234.12		+ 2462.14	+ 2481.65
		− 872.64		
15,	+ 220.25		+ 1589.50	+ 1607.85
		− 652.39		
25,	+ 195.24		+ 937.11	+ 953.38
		− 457.15		
June 4,	+ 165.00		+ 479.96	+ 493.71
		− 292.15		
14,	+ 139.56		+ 187.81	+ 199.44
		− 152.59		
24,	+ 110.88		+ 35.22	+ 44.46
		− 41.71		
July 4,	+ 77.94	− (2.74)	− (6.49)	0.00

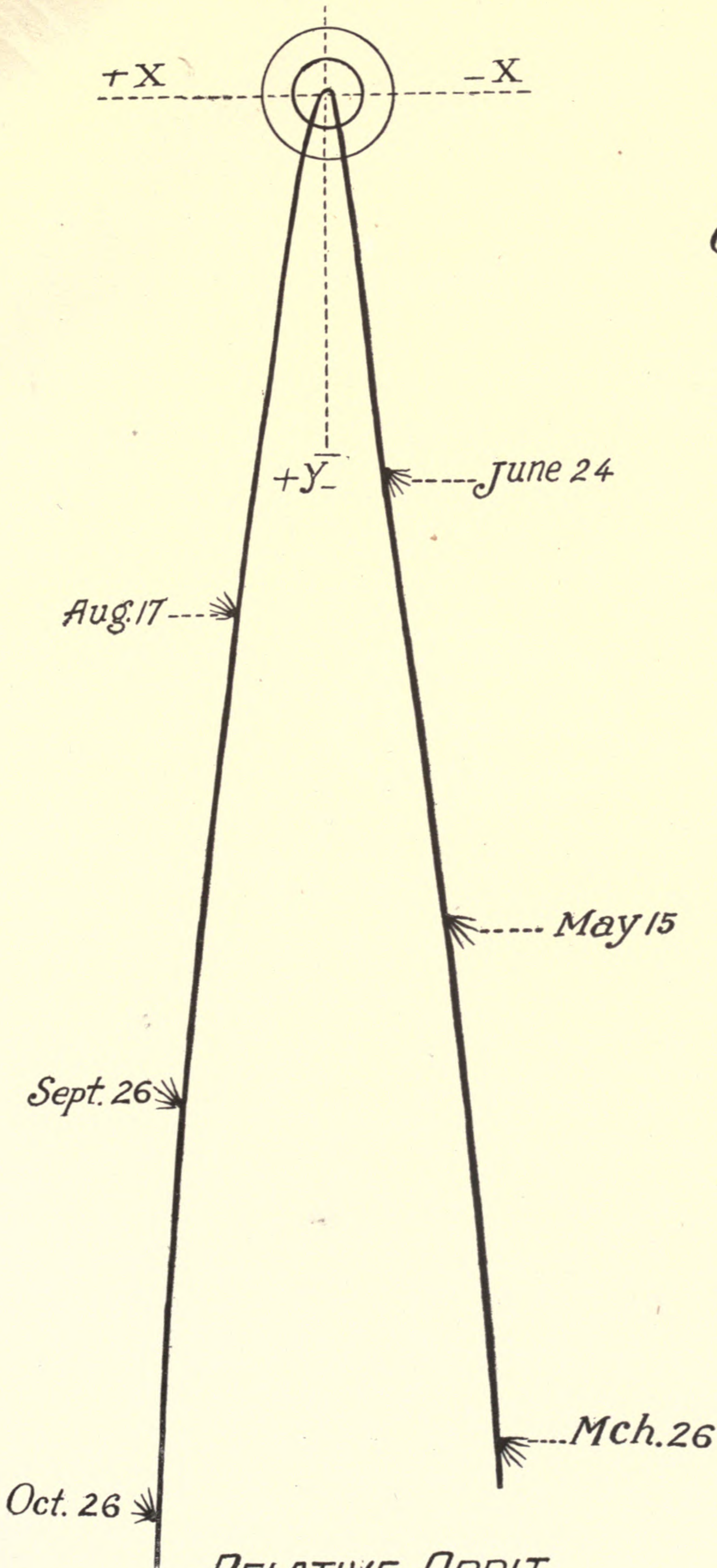
		− 7.34		
July 4,	+ 11.10		− 37.22	− 36.29
		+ 3.76		
8,	+ 6.98		− 33.46	− 32.87
		+ 10.74		
12,	+ 3.42		− 22.72	− 22.43
		+ 14.16		
16,	− 5.82		− 8.56	− 9.04
		+ 8.34		
20,	− 8.16		− 0.22	− 0.90
		+ 0.18		
24,	+ 0.50	+ (0.43)	− (0.04)	0.00
		+ 0.68		
28,	+ 2.04		+ 0.64	

	$\omega^2 \frac{d^2\delta z}{dt^2}$	$'f$	$''f$	δz
July 20,				
		− 22.02		
24,	+ 2.40		+ 65.99	+ 66.19
		− 19.62		
28,	+ 2.52		+ 46.37	+ 46.58
		− 17.10		
Aug. 1,	+ 3.06		+ 29.27	+ 29.52
		− 14.04		
5,	+ 3.84		+ 15.23	+ 15.65
		− 10.20		
9,	+ 4.67		+ 5.03	+ 5.42
		− 5.53		
13,	+ 5.50		− 0.50	− 0.04
		− 0.03		
17,	+ 6.32		− 0.53	0.00

		− 664.15		
Aug. 17,	+ 70.92		+ 2463.65	+ 2469.56
		− 593.23		
27,	+ 65.88		+ 1870.42	+ 1875.91
		− 527.35		
Sept. 6,	+ 71.52		+ 1343.07	+ 1349.03
		− 455.83		
16,	+ 81.36		+ 887.24	+ 894.02
		− 374.47		
26,	+ 92.16		+ 512.77	+ 520.45
		− 282.31		
Oct. 6,	+ 104.04		+ 230.46	+ 239.13
		− 178.27		
16,	+ 115.44		+ 52.19	+ 61.81
		− 62.83		
26,	+ 127.71	+ (1.02)	− (10.64)	

PLATE I.

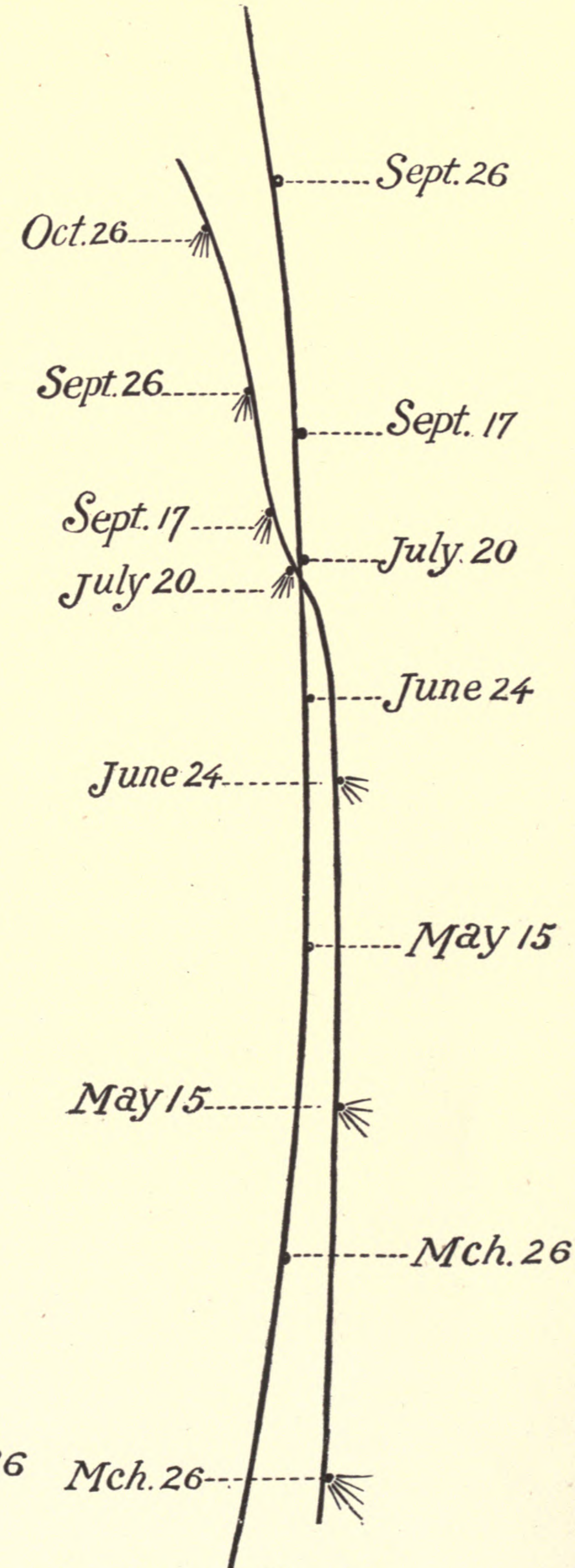
Fig. A.



RELATIVE ORBIT

Scale, 1″ = 40 radii of Jupiter.

Fig. B.

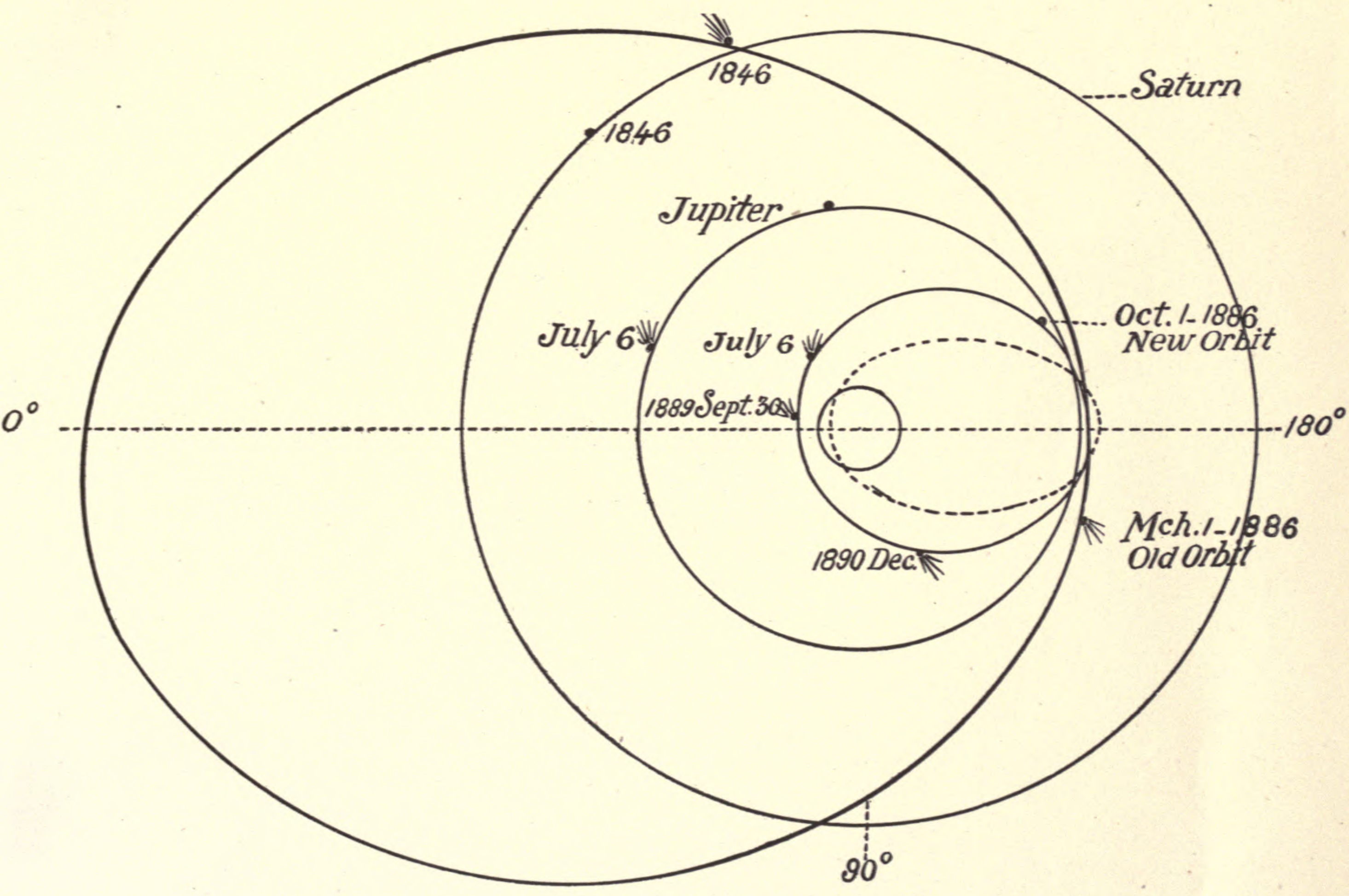


ACTUAL ORBIT

Scale, 5″ = mean distance of ⊕.

(*See page* 27.)

PLATE II.



NEW AND OLD ORBITS OF COMET V, 1889.

(*See page* 31.)